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A Model For a Large Neutrino Magnetic Transition Moment

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Abstract

We present a model in which neutrinos get a large magnetic transition moment. The model can provide a solution to the solar neutrino problem and is shown to be consistent with all experimental data. A fourth generation is necessary. Fine tuning of the neutrino masses is partially eliminated by a proper choice of symmetry. Problems with previously suggested models are discussed.

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In the last few years it has been observed that the neutrino flux measured by Davis and collaborators[1] is not only smaller than the theoretical prediction[2], but also seems to be anticorrelated with sunspot activity. Motivated by this observation, Voloshin, Vysotsky and Okun [3,4] suggested that the neutrino is a Dirac particle and has a large magnetic moment. Then, a left-handed electron neutrino produced in the core of the sun undergoes spin precession in the magnetic fields of the convective zone and emerges as a sterile, undetectable right-handed neutrino. This mechanism for depletion of the observable neutrino flux would be most efficient at times of maximal sunspot activity. Taking into account the depth of the convective zone of the sun, and the strength of the magnetic fields present there at times of maximal sunspot activity, the authors of [3,4] concluded that a magnetic moment in the range $(1-10)\cdot 10^{-11}\mu_B$ is necessary for the above scenario to work. Such large magnetic moments are hard to achieve in existing models. For example, if one adds right-handed neutrinos to the standard model, so that neutrinos become massive Dirac particles, the resulting magnetic moments produced via W loops[5]

$$\mu(\nu_e) = 3\frac{e}{8\pi^2}\frac{G_F}{\sqrt{2}}m(\nu_e) \approx 3\cdot 10^{-19}\mu_B\frac{m(\nu_e)}{1\mathrm{eV}} \tag{1}$$

are far too small.

Two years ago, Yanagida and Fukugita [6] and also Babu and Mathur [7] suggested a simple extension of the standard model for which the electron neutrino may have a magnetic moment in the desired range. They added right-handed neutrinos to the standard model, so that neutrinos became Dirac particles, and also introduced one extra charged Higgs, in the singlet representation of $SU(2)_W$. Large magnetic moments could then be produced through a loop of the new Higgs.

In this paper we will be interested in the possibility that neutrinos are Majorana particles; that is, right-handed neutrinos do not exist, or are very heavy. In this case, a magnetic interaction of the neutrinos has two peculiar characteristics. First, such an interaction violates lepton number by two units. The reason for this is that all neutrinos are left-handed and all antineutrinos are right-handed. The magnetic moment flips the helicity of the particle and therefore exchanges a neutrino for an *anti*neutrino. The other characteristic of the interaction is that it *must* flip the neutrino's flavour. The flavour change is forced by Fermi statistics, combined with the self-conjugacy of a Majorana particle. We therefore say that Majorana neutrinos have magnetic *transition* moments.

A magnetic transition moment could also be responsible for the anticorrelation of the observed neutrino flux with sunspot activity[8,9], since it will rotate a left-handed electron neutrino $(\nu_e)_L$ into a right-handed antineutrino of another flavour $(\nu_i^c)_R$. Although this antineutrino is not sterile, it can not be detected in the Davis experiment. The magnetic transition moment must again be of order of $(1-10)\cdot 10^{-11}\mu_B$. Our purpose here is to find a model for which such large magnetic transition moments exist. We first discuss the general properties of such a model — the possible choices of its Higgs structure, its symmetries and its resulting neutrino masses. We then present a specific model and show that it is consistent with all existing experimental data. Finally, we point out the theoretical and phenomenological problems associated with models that were suggested in the past.

We shall limit our search for a model to relatively simple extensions of the standard theory. We shall not extend the gauge group or allow exotic fermions; instead we will concentrate on the possibility of extending the Higgs sector. Then the magnetic transition moment should arise from the loop of a physical charged Higgs. A typical diagram is shown in figure 1. The helicity flip of the neutrino, which is necessary for the magnetic transition moment, arises from the mass operator of the internal lepton, and is denoted by the cross on one of the lepton propagators. Following the helicities of the fermions and their corresponding $SU(2)_W \times U(1)_Y$ representations, one can see that the Higgs at the vertex on the left must be an SU(2) doublet with hypercharge -1, while the Higgs at the vertex on the right must be either a triplet or a singlet, and have hypercharge -2. The physical Higgs must therefore be a *mixture* of a doublet with a singlet or a triplet (or both). The doublet-singlet mixture is a more attractive choice, since triplet Yukawa couplings are usually small, as they are related to neutrino masses. There are basically two ways to make such a physical Higgs. One is to include in the Higgs sector two doublets and one charged $SU(2)_W$ singlet. When one of the doublets develops a vacuum expectation value, the other mixes with the singlet. We call this a DDS (Doublet-Doublet-Singlet) model. So far, models for large magnetic transition moment of the neutrino have been based on DDS type Higgs sectors. Here we propose, instead, to extend the standard model by adding a charged singlet and a triplet of scalars to the standard Higgs doublet. When the doublet and the triplet develop VEVs, the physical charged scalars will include a doublet-singlet mixture. We call this a TDS (Triplet-Doublet-Singlet) model.

The next issue is what lepton number symmetry is preserved in the model. In the standard model, the lepton number of each generation is separately conserved. A model that incorporates a magnetic transition moment of Majorana neutrinos cannot have such a large symmetry, since, as mentioned above, the magnetic interaction itself breaks lepton number by two units. However, we propose to keep a subgroup of the standard model symmetry. Such a subgroup will benefit us in two ways. It will enable us to avoid some of the fine tuning that is necessary in the neutrino mass matrix, and will also forbid some rare processes which usually occur when lepton number is broken.

There are two fine-tuning problems with neutrino masses: The first occurs in every model with a large magnetic (transition) moment[10]. These models will, in general, also give large loop-corrections to neutrino masses, because generic diagrams that contribute to neutrino magnetic moments, such as the one in figure 1, become mass-corrections when the photon line is removed. The finite part of the mass correction is related to the magnetic moment through

$$\frac{\delta m}{\mu} \sim \frac{M^2}{e}\,, \tag{2}$$

where M is the mass of the heavier particle in the loop, and e is the electric charge. Since $M \gtrsim 20\text{GeV}$ † and we seek $\mu \sim (1-10)\cdot 10^{-11}\mu_B$, this gives $\delta m \gtrsim$ few keV. This means that it is necessary to fine-tune the mass by at least two orders of magnitude to keep it below the experimental bound of 18eV.

The second fine-tuning problem problem is potentially much more severe, and occurs only in models for magnetic *transition* moments. The magnetic interaction in this case involves two neutrino flavours, and it will be able to flip one into the other only if the mass difference between them is extremely small. To see this, consider the Hamiltonian that describes the physics of the neutrino in the convective zone of the sun:

$$H = k + \begin{pmatrix} \frac{m^2(\nu_e)}{2E} + V(\nu_e) & \mu B \\ \mu B & \frac{m^2(\nu_i^c)}{2E} + V(\nu_i^c) \end{pmatrix}\,. \tag{3}$$

Here the neutrino is treated as a quantum mechanical system with two possible states: left-handed electron neutrino and right-handed antineutrino of flavour $i \neq e$. k and E are the momentum and energy of the neutrino, $m(\nu_e)$ and $m(\nu_i^c)$ are the masses of the two neutrino states and $V(\nu_e)$ and $V(\nu_i^c)$ are the effective potentials produced by coherent weak interaction with the background matter [15] The magnetic moment interaction μB will be able to rotate ν_e to the undetectable ν_i^c only if the energy splitting between the two neutrino states is not much bigger than the magnetic energy:

$$\left|\frac{\Delta m^2}{2E} + \Delta V\right| \lesssim |\mu B|\,. \tag{4}$$

At times of maximal sunspot activity $|\Delta V| \leq |\mu B|$ through most of the convective zone [4]. It is therefore sufficient to impose:

$$\left|\frac{\Delta m^2}{2E}\right| \lesssim |\mu B|\,. \tag{5}$$

† If the heavier particle is the charged Higgs, its mass is larger than 19 GeV [11]. Tristan can improve this bound to 25 GeV [12]. If the heavier particle is a new charged lepton, its mass is larger than 28 GeV [13].

As the energy E of the neutrinos that are detected in Davis experiment is a few MeV, the magnetic transition moment is $\leq 1.5 \cdot 10^{-10}\mu_B$ [16], and the strength of the magnetic fields at times of maximal activity is $1-10$ kiloGauss, this implies [4]:

$$\Delta m^2 \leq 10^{-7}\mathrm{eV}^2\ . \tag{6}$$

Thus, in this case, fine-tuning over many more orders of magnitude will be required.

The alternative to fine-tuning is to incorporate some symmetry into the model to protect neutrino masses. The first attempt to do this was by Voloshin[17], who proposed an SU(2) symmetry which would force the neutrino masses to vanish. However, this symmetry is broken in nature, and attempts to use it have not been successful. (We will discuss such a model by Babu and Mohapatra [18] later in this paper.) Here we choose to conserve only the U(1) symmetry generated by $N_e - N_i$, the difference between electron lepton number and the lepton number of the i'th generation. Although this does not protect the neutrino masses, it does force the neutrino mass-squared difference to vanish [19], alleviating the much more serious fine-tuning problem of eq. (6). This symmetry is a U(1) subgroup of Voloshin's SU(2). However, unlike the full SU(2), the subgroup can clearly be left unbroken. We note that our choice of symmetry to protect Δm^2 is not unique. We could, for example, choose the U(1) generated by $N_e - N_i - N_j$, and could also use only discrete Z_4 subgroups, instead of the full U(1)'s.

This choice of symmetry also solves a potential problem with rare processes. For example, the decays $\mu \to e\gamma$ and $\mu \to eee$ are absolutely forbidden, even if $i = \mu$, even though in this case electron and muon number are both separately violated. Similarly, neutrinoless double-beta decays are also forbidden.

We now have all the ingredients to write down the Lagrangian for our model. We denote the singlet, doublet, and triplet Higgs fields by η, ϕ, and χ respectively:

$$\begin{aligned} \eta &= \eta^- \\ \phi &= \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \\ \chi &= \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^- & -\chi^0 \\ \chi^{--} & -\frac{1}{\sqrt{2}}\chi^- \end{pmatrix}\ . \end{aligned} \tag{7}$$

Note that all the Higgs particles must have vanishing $N_e - N_i$. The doublet and the triplet should not carry any $N_e - N_i$ because they develop VEVs, and we do not wish to break the symmetry. The singlet too cannot carry any $N_e - N_i$, since it must mix with the doublet that carries zero $N_e - N_i$. It follows that the Yukawa interactions of the e and i lepton families are completely separated from the interactions of all the other families. We are therefore able to ignore the other leptons and treat the problem in a two-generation framework. The Yukawa interaction of the leptons is given by:

$$\begin{aligned} \mathcal{L}_Y &= y_e\, \bar{e}_R \phi^t i\tau_2 L_e + y_i\, \bar{l}_{iR} \phi^t i\tau_2 L_i + \text{h.c.} \\ &+ y_T\, L_e^t C^\dagger i\tau_2 \chi^\dagger L_i + \text{h.c.} \\ &+ y_S\, L_e^t C^\dagger i\tau_2 L_i \eta^\dagger + \text{h.c.} \\ &+ \text{terms involving other leptonic generations}\,, \end{aligned} \tag{8}$$

where C is the charge conjugation matrix for the Dirac spinors, and $L_j = \binom{\nu_j}{l_j}_L$ is the j'th leptonic doublet. As usual, all the doublet couplings are diagonal in flavour space, since it is the only doublet in the model and it therefore defines the mass eigenstates. The triplet couplings must be off-diagonal, because of the $N_e - N_i$ conservation. Therefore, when the triplet develops a VEV, the mass matrix of ν_e and ν_i will have only off-diagonal terms. Since the mass matrix is symmetric, it is proportional to σ_1. The mass-squared matrix is then proportional to 1, and $m^2(\nu_e) = m^2(\nu_i)$, as desired.

The Higgs potential can be written as

$$\begin{aligned} V &= \lambda_1(\phi^\dagger\phi - v^2)^2 \\ &+ \lambda_2[\mathrm{Tr}(\chi^\dagger\chi) - (\gamma v)^2]^2 \\ &+ \lambda_3\, \mathrm{Tr}[(v\chi + \gamma\phi\phi^t i\tau_2)(v\chi^\dagger - \gamma i\tau_2\phi^*\phi^\dagger)] \\ &+ \lambda_4[(\eta^\dagger\eta)^2 + M^2\eta^\dagger\eta] \\ &+ \lambda_5(\phi^\dagger\phi - v^2)(\mathrm{Tr}(\chi^\dagger\chi) - (\gamma v)^2) \\ &+ \lambda_6(\phi^\dagger\phi - v^2)\eta^\dagger\eta \\ &+ \lambda_7[\mathrm{Tr}(\chi^\dagger\chi) - (\gamma v)^2]\eta^\dagger\eta \\ &+ \lambda_8\, \mathrm{Tr}(\chi\chi)\, \mathrm{Tr}(\chi^\dagger\chi^\dagger) \\ &+ \lambda_9[\phi^\dagger\phi\, \mathrm{Tr}(\chi^\dagger\chi) - \phi^\dagger\chi\chi^\dagger\phi] \\ &+ \lambda_{10}[e^{i\delta}\eta\eta + \mathrm{Tr}(\chi\chi)][e^{-i\delta}\eta^\dagger\eta^\dagger + \mathrm{Tr}(\chi^\dagger\chi^\dagger)] \\ &+ \lambda_{11}[\eta\phi^t + \phi^t\chi + \gamma v\phi^t i\tau_2][\phi\eta^\dagger + \chi^\dagger\phi - \gamma v i\tau_2\phi^*]\,. \end{aligned} \tag{9}$$

Here, all the parameters are real. $\lambda_1 ... \lambda_4$ and $\lambda_8 ... \lambda_{11}$ are positive, $|\lambda_5| \leq \sqrt{2\lambda_1\lambda_2}$, $|\lambda_6| \leq \sqrt{2\lambda_1\lambda_4}$ and $|\lambda_7| \leq \sqrt{2\lambda_2\lambda_4}$. δ is a phase, and M and v have the dimensions of masses, and are chosen to be positive. The Higgs potential is manifestly nonnegative. It vanishes at the point:

$$\langle\eta\rangle = 0 \qquad \langle\phi\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} \qquad \langle\chi\rangle = \begin{pmatrix} 0 & -\gamma v \\ 0 & 0 \end{pmatrix}\,, \tag{10}$$

which is, therefore, its absolute minimum.

The parameter λ_{11} produces the desired doublet-singlet mixing. When the doublet and the triplet develop their vacuum expectation values, the term $\lambda_{11}(\eta\phi^\dagger\chi^\dagger\phi + \text{h.c.})$ becomes $\lambda_{11}(\eta^-\phi^+\langle\chi^0\rangle\langle\phi^0\rangle) + \ldots$. Since the magnetic transition moment depends on this mixing, we must require that λ_{11} is not small.

Some comments on the parameter γ should be made here. First, γ is the lepton number breaking parameter in our model. To see this, note that the Yukawa potential implies that χ and η each carry two units of lepton number, while ϕ carries none. Then, lepton number is explicitly broken in the Higgs potential by the terms $\gamma v(\lambda_3 + \lambda_{11})\,\phi^t i\tau_2\chi^\dagger\phi + \text{h.c.}$, and is also broken in the vacuum by the VEV $\langle\chi^0\rangle = \gamma v$. All lepton number breaking, explicit and by VEV, is therefore linear in γ. Note that γ must be small, since the VEV of the triplet will change the Weinberg mass relation. Our present knowledge of the ρ parameter implies that $\gamma \leq 0.07$ [11]. There is no problem of fine-tuning to keep γ small, as it is protected by the approximate lepton number symmetry. Also, note that there is no pseudo-Goldstone Majoron-like particle associated with the breakdown of lepton number by the triplet VEV [14]. This can be seen by considering the $\gamma \to 0$ limit: In this limit there is no Goldstone boson, since lepton number is conserved by the Lagrangian and also by the vacuum. Therefore, in the case where γ is small but not vanishing, there is no pseudo-Goldstone boson. One may verify by explicit calculation that none of the Higgs bosons have masses which are proportional to γ. We conclude that having γ be the lepton number breaking parameter of our model and requiring it to be small is satisfactory from theoretical and phenomenological points of view. There is however one crucial disadvantage. Since the magnetic transition moment is a lepton number breaking quantity, it must be proportional to γ. As we will see, the smallness of γ will make it impossible to achieve μ of the order of $10^{-11}\mu_B$ unless a fourth generation is added.

The physical Higgs spectrum includes one doubly-charged scalar, two singly-charged scalars and three neutral particles. The magnetic transition moment will be induced by the scalars that carry one unit of electromagnetic charge, and we will therefore present their masses and mixings in detail. We denote by G^- the Goldstone particle which is to constitute the longitudinal component of W^-

$$G^- = (\phi^- + \sqrt{2}\gamma\chi^-)\,. \tag{11}$$

The other combination of ϕ^- and χ^- we call ϕ_m^-

$$\phi_m^- = -\sqrt{2}\gamma\phi^- + \chi^-\,, \tag{12}$$

where we have neglected corrections of order γ^2; in the following, we shall continue to ignore terms of this order. ϕ_m^- mixes with the singlet η^-, with a mass matrix that is given by:

$$\begin{pmatrix} \phi_m^- & \eta^- \end{pmatrix} \begin{pmatrix} (2\lambda_3+\lambda_9+\lambda_{11})\frac{1}{2}v^2 & \lambda_{11}\sqrt{\frac{1}{2}}v^2 \\ \lambda_{11}\sqrt{\frac{1}{2}}v^2 & \lambda_4 M^2 + \lambda_{11}v^2 \end{pmatrix} \begin{pmatrix} \phi_m^+ \\ \eta^+ \end{pmatrix} . \tag{13}$$

Note that the mixing is generated by λ_{11}, as promised. We denote the mass eigenstates by H_1 and H_2, their masses by M_1 and M_2, and the mixing angle by θ:

$$\begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_m^- \\ \eta^- \end{pmatrix} , \tag{14}$$

where

$$\tan 2\theta = \frac{\sqrt{2}\lambda_{11}}{\lambda_3 + \frac{1}{2}(\lambda_9 - \lambda_{11}) - \lambda_4 \frac{M^2}{v^2}} . \tag{15}$$

The scalars H_I have the doublet-singlet mixture which is necessary for producing the magnetic transition moment. For example,

$$H_1 = \cos\theta \left(\chi^- - \sqrt{2}\gamma\phi^-\right) + \sin\theta\eta^- . \tag{16}$$

The coefficient of the doublet component in H_1 is $\sqrt{2}\gamma\cos\theta$, and that of the singlet is $\sin\theta$. The contribution of H_1 to the magnetic transition moment will therefore be proportional to $\gamma\cos\theta\sin\theta$, as will the contribution of H_2.

We can now evaluate the magnetic transition moment of the e and i neutrinos. We find:

$$\mu(\nu_e,\nu_i) = \frac{e}{16\pi^2}\frac{g_W}{M_W} y_S\,\gamma\sin\theta\cos\theta\left\{[g(x_2^i) - g(x_1^i)] + [g(x_2^e) - g(x_1^e)]\right\} , \tag{17}$$

where g_W is the SU(2) weak coupling constant $x_I^j = m^2(l_j)/M_I^2$ for $j = e$ or i, $I = 1,2$, and the function $g(x)$ is

$$g(x) = \frac{x}{1-x}\left[\frac{1}{1-x}\log(\frac{1}{x}) - 1\right] . \tag{18}$$

$g(x_I^j)$ is proportional to $m^2(l_j)$ — one power of the lepton mass coming from the doublet Yukawa coupling, and the other from the spin-flip operator. The contribution of the diagrams with internal electrons are therefore negligible, and we will ignore them from now on. We have also neglected contributions proportional to the triplet coupling, since, as argued before, this coupling is small.

At this point, we can decide what the i'th generation should be. We rewrite eq. (17) in the form

$$\mu(\nu_e,\nu_i) = 1.75\cdot 10^{-9}\mu_B\, y_S\,\frac{\gamma}{0.07}\sin 2\theta\,\Delta g , \tag{19}$$

where $\Delta g = g(x_1^i) - g(x_2^i)$. Since we wish to achieve $|\mu| > 10^{-11}\mu_B$, we require

$$\left| y_S\,\frac{\gamma}{0.07}\sin 2\theta\,\Delta g\right| \geq 0.0057 . \tag{20}$$

The muon cannot obey the above requirement. To see this, note that $|\Delta g| \leq \max_I g(x_I^\mu)$, since g is positive. Assuming that the charged Higgs is not lighter than 20 GeV, we find:

$$\max_I [g(x_I^\mu)] \leq g(x^\mu)\Big|_{M_H=20GeV} \simeq 2.7 \cdot 10^{-4} , \tag{21}$$

since g is monotonically increasing. Therefore, the muon cannot satisfy (20).

The only other possibility among the known leptons is the τ, and it, too, is excluded. To show this, we will need a more elaborate argument than the one used in the case of the μ. Consider the contributions of H_1 and H_2 to leptonic τ decays. The contributions depending on the doublet component of these Higgs particles are proportional to either $\gamma m_e/M_W$ or $\gamma m_\tau/M_W$ and are hence negligible. The singlet component contributes only to τ decay to $e\nu\bar{\nu}$, and therefore e–μ universality in τ decay is destroyed:

$$\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mu\bar{\nu}_\mu\nu_\tau)} = \left[\frac{\frac{g_W^2}{M_W^2} - y_S^2\left(\frac{\sin^2\theta}{M_1^2} + \frac{\cos^2\theta}{M_2^2}\right)}{\frac{g_W^2}{M_W^2}}\right]^2 F , \tag{22}$$

where $F \simeq 1.027$ is the ratio of the phase-space factors. At present, leptonic τ decays are consistent with universality [11]

$$\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mu\bar{\nu}_\mu\nu_\tau)} = (0.960 \pm 0.033)F . \tag{23}$$

Therefore, we conclude that

$$\frac{y_S^2}{g_W^2} M_W^2 \left(\frac{\sin^2\theta}{M_1^2} + \frac{\cos^2\theta}{M_2^2}\right) \leq 0.037 . \tag{24}$$

Using the last inequality, and assuming $M_I \gtrsim 20$ GeV we can show that

$$|y_S \sin 2\theta \Delta g| \leq 0.002 , \tag{25}$$

so the τ also cannot satisfy (20). The conclusion is therefore that l_i must be a fourth generation lepton, which we will call ℓ. It is easy to satisfy (20) for ℓ, since this lepton can be as heavy as the Higgs particles or even heavier. For example, if $x_1^\ell = 2$ and $x_2^\ell = 1/2$ then $\Delta g = 0.23$, and (20) can then clearly be satisfied for a wide range of values of y_S, θ and γ. Now, that we are able to produce large magnetic transition moment, we should also recall the upper limit on the magnetic transition moment. The present experimental bound on $\mu(\nu_e, \nu_\ell)$ is $1.5 \cdot 10^{-10}\mu_B$ [16], implying that:

$$0.0057 \leq \left| y_S \frac{\gamma}{0.07} \sin 2\theta \Delta g \right| \leq 0.085 . \tag{26}$$

One could think that phenomenological constraints on the parameters of our model could lead to difficulties in satisfying the inequalities (26) even in the case of a fourth generation lepton. We will now show that such difficulties do not arise. First, as described above, lepton-number violating processes like $\mu \to e\gamma$, $\mu \to eee$ or neutrinoless double beta decay do not occur in our model. Also, the $(g-2)$ of the electron and the muon do not present any difficulty. Of course, contributions to $(g-2)$ do arise from loops of the physical Higgs particles. However, the doublet and triplet components of the physical scalars will give negligible contribution since their Yukawa couplings are small. Since the singlet component couples only to left-handed particles, while $(g-2)$ is associated with a spin flipping operator that involves a left-handed and a right-handed particle, one can show that its contribution is also sufficiently small. The only processes which can give significant bounds on the parameters of our model are $\nu_\ell - e$ elastic scattering and leptonic decays of the π and K mesons. All of these have new contributions, mediated by H_1 and H_2, which we will now discuss in detail.

$\nu_\ell - e$ elastic scattering: The new contribution to $\nu_\ell - e$ elastic scattering is described in figure 2. When ν_ℓ passes through matter, this contribution may have an important effect on its coherent forward scattering. In particular, we find that the Hamiltonian of the $(\nu_e)_L - (\bar{\nu}_\ell)_R$ system in the sun is modified to:

$$H = k + \frac{m^2}{2E} + \begin{pmatrix} V(\nu_e) & \mu B \\ \mu B & V(\bar{\nu}_\ell) \end{pmatrix} , \tag{27}$$

where, now,

$$\Delta V = \frac{g_W^2}{4M_W^2}(n_e - n_n) + \frac{y_S^2}{4}\left(\frac{\sin^2\theta}{M_1^2} + \frac{\cos^2\theta}{M_2^2}\right) n_e \,. \tag{28}$$

n_e and n_n are the number densities of the electrons and neutrons respectively. As we discussed previously, significant depletion of the detectable neutrino flux will occur only if $|\Delta V| \leq |\mu B|$ through most of the convective zone. The standard model piece of the potential difference obeys this condition. We will require that the new contribution to ΔV does not exceed the standard one. Neglecting corrections of order n_n/n_e (this is justified in the convective zone where $n_e \approx 6n_n$) this implies that

$$y_S^2\left(\frac{\sin^2\theta}{M_1^2} + \frac{\cos^2\theta}{M_2^2}\right) \lesssim \frac{g_W^2}{M_W^2} \,. \tag{29}$$

Rewritten in terms of x_1^ℓ, x_2^ℓ and m_ℓ, this inequality becomes

$$y_S^2(x_1^\ell \sin^2\theta + x_2^\ell \cos^2\theta) \lesssim g_W^2 \frac{m_\ell^2}{M_W^2} \,. \tag{30}$$

Leptonic decays of π and K mesons: Our model predicts that the π and K decay to $e\,\nu_\ell$ through H_1 or H_2 exchange (see figure 3). At first sight, one could think that this new decay mode could not compete with the standard decay to $e\bar{\nu}_e$, since H_1 and H_2 can couple to quarks only through their doublet component. This component is suppressed by a γ factor and, in addition, its couplings to quarks are typically of the form $g_W\, m_q/M_W$ where m_q is the mass of the up, down or strange quark. The amplitude for π or K decay to $e\,\nu_\ell$ is therefore suppressed by $\gamma\, m_q/M_W$. Recall, however, that the standard amplitudes for π and K decays to $e\,\bar{\nu}_e$ are helicity suppressed, because of the current-current character of the weak interaction. This suppression factor is m_e/Λ where Λ is a QCD scale. In practice, one does not distinguish the $e\,\bar{\nu}_e$ mode from an $e\,\nu_\ell$ mode in a π or K decay experiment, since the neutrino is not directly detected. The effect of the new mode is therefore to enhance the decay rate of a pion or kaon to electron and neutrino. At present these decay rates are consistent with the standard model prediction, which is based on universality and the observed muonic decay rates. We must, therefore, require that the decay rate to the new mode does not exceed the experimental error [11]:

$$\frac{\Gamma(\pi \to e\,\nu_\ell)}{\Gamma(\pi \to e\bar{\nu}_e)} = \left(\frac{y_S\gamma \sin\theta \cos\theta \frac{g_W}{\sqrt{2}} \frac{m_\pi^2}{M_W} \left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right)}{\frac{g_W^2}{2M_W^2} m_e} \right)^2 \leq 0.02 \,. \tag{31}$$

Note that the combination $y_S\gamma \cos\theta \sin\theta$ that appeared in the expression for the magnetic transition moment appears also in the amplitude for $\pi \to e\,\nu_\ell$ decay. The reason for this is that, in both cases, the H_I scalars couple at one vertex through their doublet component and at the other vertex through their singlet component. The last inequality implies:

$$\left| y_S \frac{\gamma}{0.07} \sin 2\theta \, \frac{M_W^2}{m_\ell^2} \Delta x^\ell \right| \leq 3.8 \,. \tag{32}$$

The similar bound from K decays is stricter:

$$\left| y_S \frac{\gamma}{0.07} \sin 2\theta \, \frac{M_W^2}{m_\ell^2} \Delta x^\ell \right| \leq 0.47 \,. \tag{33}$$

There is a wide range of parameters for which (26), (30) and (33) are simultaneously satisfied. For example, if we fix $x_1^\ell = 2$, $x_2^\ell = 1/2$, $\gamma = 0.07$ and $\theta = 45°$, and use the lower bound on the mass of a possible fourth generation lepton of $m_\ell > 28\mathrm{GeV}$ [13], the allowed region of m_ℓ–y_S space is shown in figure 4.

We will now study the mass of the neutrino in our model. The diagrams that contribute to the magnetic transition moment $\mu(\nu_e, \nu_\ell)$ become mass-corrections when the photon line is removed, giving:

$$\delta m(\nu_e, \nu_\ell) = \frac{1}{16\pi^2} \frac{g_W}{M_W} \gamma \sin\theta \cos\theta y_S \left\{ m_\ell^2 [f(x_2^\ell) - f(x_1^\ell)] - m_e^2 [f(x_2^e) - f(x_1^e)] \right\} \,, \tag{34}$$

where $m(\nu_e, \nu_\ell)$ is the off-diagonal term in the neutrino mass matrix (diagonal terms are forbidden by $N_e - N_\ell$ conservation). $f(x)$ is defined by

$$f(x) = \frac{1}{1-x} \log\left(\frac{1}{x}\right) . \tag{35}$$

The contribution of the loops with internal electron lines are again proportional to two powers of the electron mass, and are negligible. Comparing the mass correction to the magnetic transition moment, we find:

$$\left|\frac{\delta m}{\mu}\right| = \frac{m_\ell^2}{e} \left|\frac{\Delta f}{\Delta g}\right| . \tag{36}$$

By comparing the derivatives of f and g, one can see that $|\Delta f| > |\Delta g|$ for all x_1^ℓ and x_2^ℓ. Eq. (36) then implies that

$$\left|\frac{\delta m}{\mu}\right| \geq \frac{m_\ell^2}{e} . \tag{37}$$

Since $m_\ell > 28\text{GeV}$, and our purpose is to produce $\mu \gtrsim 10^{-11} \mu_B$,

$$|\delta m| > 8\text{keV} . \tag{38}$$

To achieve a neutrino mass compatible with the experimental upper bound, one must fine-tune this loop-correction against the tree-level term. We thus have a fine-tuning of about three orders of magnitude. While this is aesthetically unpleasing, it appears to be necessary. A similar fine-tuning is also needed in the models [6,7] for a Dirac neutrino with a large magnetic moment.

Finally, we will briefly discuss other proposed models for large magnetic transition moments. All these models have a DDS Higgs structure.

Babu and Mathur suggested a minimal DDS model [7]. Their Higgs spectrum contains two doublets ϕ_1 and ϕ_2, and one charged singlet η. Both ϕ_1 and ϕ_2 develop VEVs, v_1 and v_2. One linear combination of ϕ_1^- and ϕ_2^- is eaten by the W^-, and the other is a physical Higgs and mixes with the η^-. The mass eigenstates, H_1 and H_2, are doublet-singlet mixtures, and induce a magnetic transition moment. Since the authors impose a discrete symmetry, allowing only ϕ_1 and η to couple to fermions, one finds that the expression for the magnetic transition moment is almost identical to the expression in our model. The only difference is that "γ" should be replaced by v_1/v_2. An advantage of their approach is that, unlike γ, the ratio v_1/v_2 is not necessarily small, and one can produce a large magnetic transition moment using a τ in the loop. The problem with this model appears in the corrections to the neutrino mass. The expression for this is again the same as our expression after replacing γ by v_1/v_2. Therefore, again,

$$\left|\frac{\delta m}{\mu}\right| = \frac{m_\tau^2}{e} \left|\frac{\Delta f}{\Delta g}\right| . \tag{39}$$

Since $M_H \gg m_\tau$, $\left|\frac{\Delta f}{\Delta g}\right|$ is $(M_H/m_\tau)^2$, up to logarithmic factors. Indeed, by studying the derivatives of f and g, one can show that for $M_H > 20\text{GeV}$, $|\Delta f^\tau| > 40|\Delta g^\tau|$. If $\mu \geq 10^{-11}\mu_B$, the one-loop mass correction δm is greater than 1.25keV. However, unlike the case of TDS models, there is no tree-level mass, and one can not bring the neutrino mass down below its experimental bound.

The other model we will consider was suggested by Babu and Mohapatra [18]. This model attempts to exploit Voloshin's SU(2), in order to get acceptably small neutrino masses without fine-tuning. In this case, the SU(2) acts horizontally between the first and second leptonic generations. Since horizontal symmetries are broken in nature, the authors proposed to gauge the SU(2), and then break it spontaneously at ~ 1 TeV. A serious problem occurs immediately. The $\text{SU}(2)_H$, has a global anomaly, since there is an odd number of fermion doublets: $\begin{pmatrix} e^- \\ \mu^- \end{pmatrix}_L$, $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L$ and $\begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix}_L$. Even if this could be fixed, there is a serious phenomenological problem arising from the symmetry structure of the model. The authors imposed a discrete symmetry, in order to make the Yukawa coupling h_1 vanish (the reader is referred to the original paper for notation). However, if h_1 is set to zero, the Lagrangian possesses an unexpected U(1) symmetry. The U(1) charges are:

$$Q(\psi_R, \psi_{3L}, \psi_{3R}, \Phi, \eta) = (2, 1, 1, -1, -1) . \tag{40}$$

This U(1) is spontaneously broken when the Higgs fields develop their VEVs. The theory therefore contains a Goldstone boson, which is phenomenologically unacceptable. For example, it gives a large correction to the $(g-2)$ of the muon. The obvious cure for this problem is to break the symmetry explicitly. However, this also destroys the good features of the model. The authors noted that undesirable lepton-violating processes do not occur at tree level, and that $\Delta m^2 = 0$ to one loop. This is because the broken U(1) symmetry leaves a remnant Z_4 group which is generated by

$$Z = \mathrm{e}^{-i\pi Q/2} \mathrm{e}^{i\pi T_3} S , \tag{41}$$

where T_3 is the diagonal generator of $\text{SU}(2)_H$, and S acts only on the σ_2 field; $S(\sigma_2) = -\sigma_2$. In the leptonic sector, the generator Z reduces to

$$Z|_{\text{leptonic sector}} = \mathrm{e}^{i\pi(N_e - N_\mu - N_\tau)/2} , \tag{42}$$

so that this Z_4 symmetry plays the role of $\text{U}(1)_{N_e - N_\ell}$ in our model. One cannot modify their model to remove the Goldstone boson, without destroying the Z_4.

In general, it appears difficult to utilize Voloshin's SU(2) symmetry to protect neutrino masses [20]. This is because the symmetry is not respected in nature, and it must be broken far above the weak scale. The neutrino typically gets its mass from physics of near or below the weak scale. The neutrino mass is therefore unlikely to be protected by a symmetry it cannot see. For example, in the model of Babu and Mohapatra, the λ_4 term in the Higgs potential feeds the breaking of $SU(2)_H$ down to the Higgs boson masses matrix, destroying the near equality of the $\eta_1 - \phi_1$ and $\eta_2 - \phi_2$ mass matrices necessary to keep the neutrino light.

In conclusion, we have presented an extension of the standard model, in which there is a magnetic transition moment between the electron neutrino and a fourth generation neutrino of the order of $(1-10)\cdot 10^{-11}\mu_B$. This is large enough to solve the solar neutrino problem, including the anticorrelation of the neutrino flux and sunspot activity. In addition to containing a fourth generation, the model has two extra Higgs multiplets — a triplet and a charged singlet. Previous models, which were based on extra singlets and doublets, suffer from theoretical and phenomenological difficulties. Our choice of Higgs sector allows us to produce this magnetic transition moment with acceptable neutrino masses, although the latter does require a fine-tuning of order 10^{-3}. The model is invariant under the U(1) generated by the difference of electron lepton number and fourth generation lepton number. This forces the mass-squared difference of the two neutrinos to vanish, enabling the magnetic fields in the convective zone of the sun to rotate one neutrino to the other. The symmetry also forbids unpleasant processes which usually appear in models with broken lepton number. Our model is therefore consistent with present phenomenology for a large range of parameters.

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Figure Captions

Figure 1: A typical contribution to the neutrino magnetic transition moment. The helicity on the fermion line is flipped at the Yukawa vertices and by the mass operator (denoted by a cross on one of the internal lepton propagators). With the photon leg removed, this diagram contributes to the neutrino mass.

Figure 2: Contributions to $\nu_\ell - e$ elastic scattering mediated by the scalars H_1^- and H_2^-.

Figure 3: Contributions to leptonic decays of π or K mediated by the scalars H_1^- and H_2^-.

Figure 4: Allowed region of $m_\ell - y_S$ space allowed by the experimental constraints. Here we have taken $x_1^\ell = 2$, $x_2^\ell = 1/2$, $\gamma = 0.07$, and $\theta = 45°$. The numbers in parentheses label the curves by equation number.

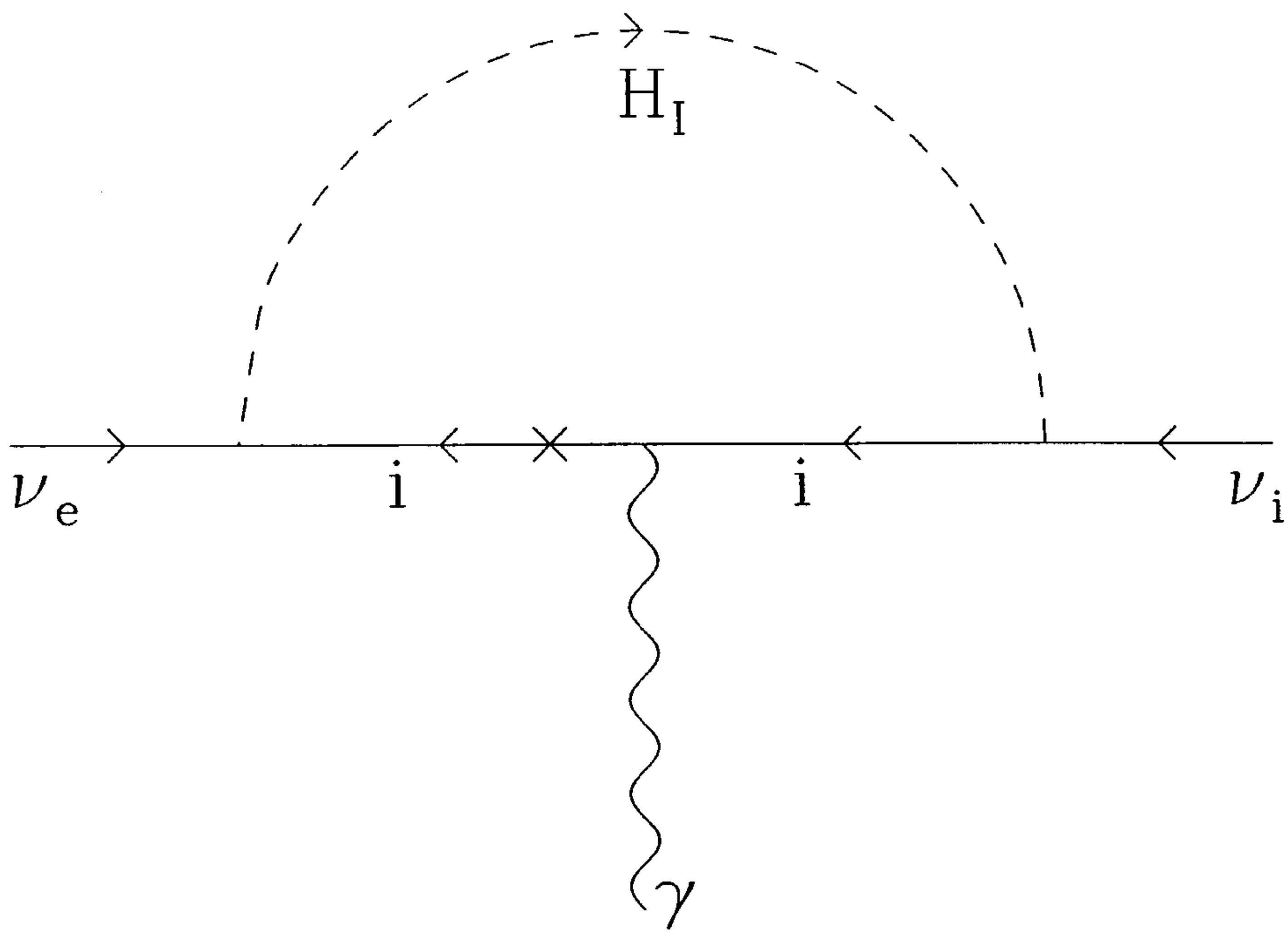


Fig 1

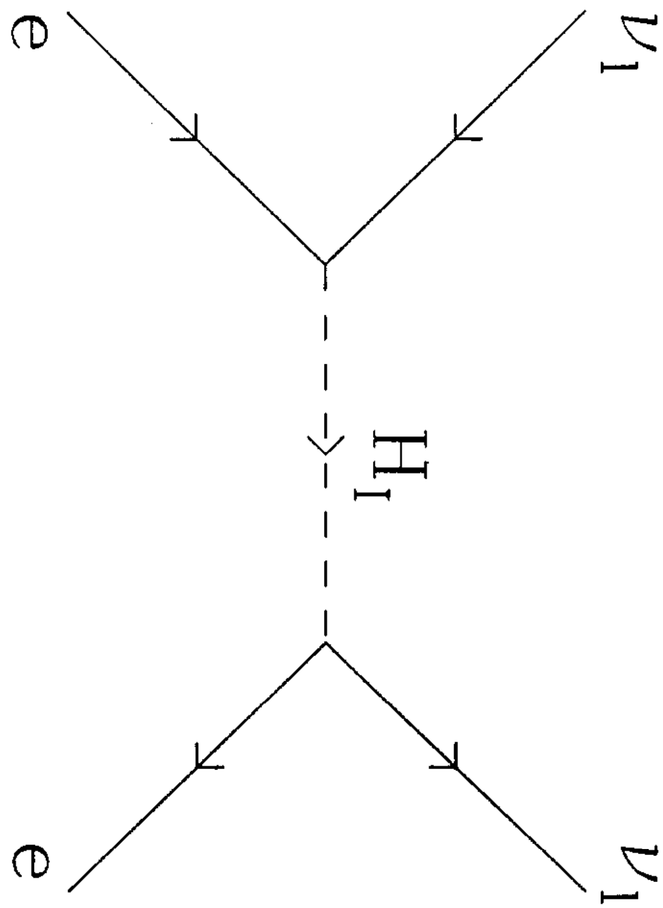


Fig. 2

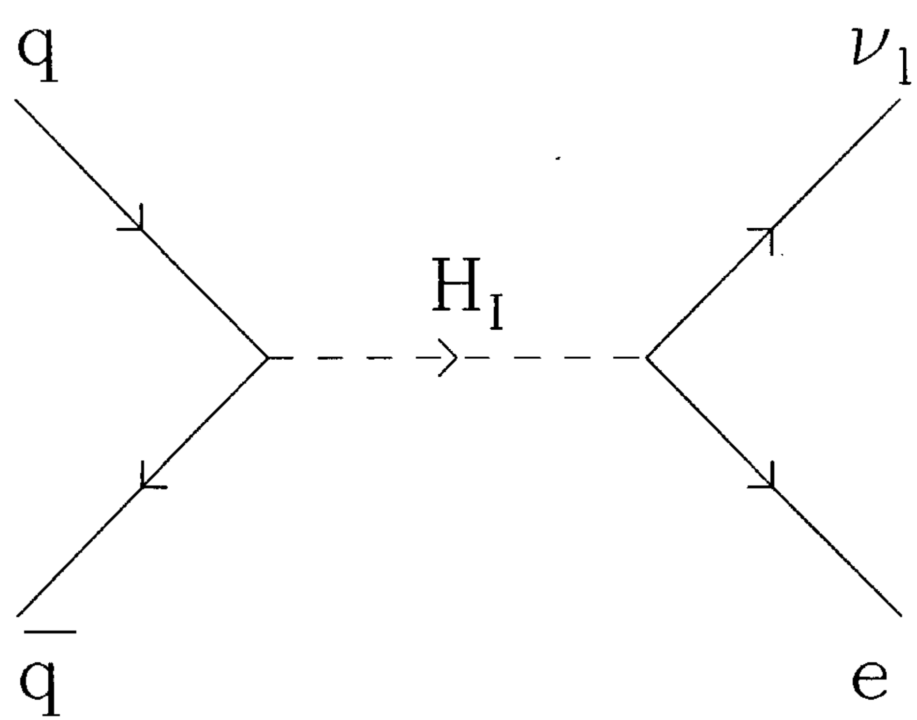
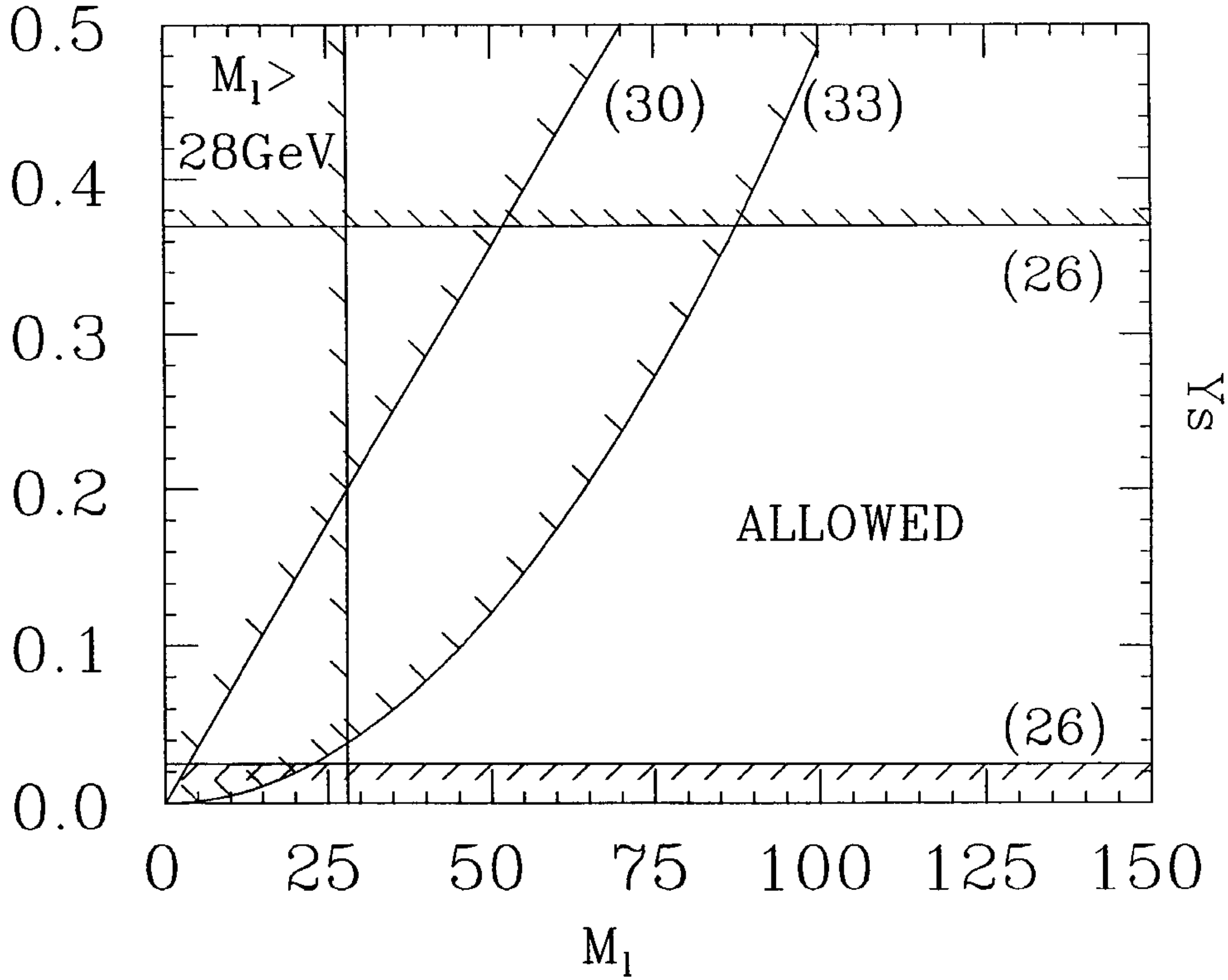


Fig. 3



Ys
$M_l >$ 28GeV
(30)
(33)
(26)
ALLOWED
(26)
0.5
0.4
0.3
0.2
0.1
0.0
0
25
50
75
100
125
150
M_l

Fig. 4